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Abstract

This note shows that permanent fluctuations in the Cobweb model—though inconsistent with a rational expectations equilibrium—can be justified as being rational when reinterpreting the model in the theory of rational beliefs.

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1. Introduction. The Cobweb model of output and price fluctuations attempts to explain business cycles within a simple dynamic supply–demand model. When supply adjustment is slow (as e.g. in agriculture) and suppliers base their production decision on the last observed price, cycles of fixed or growing amplitude can occur, see e.g. Ezekiel (1938). Static as well as adaptive expectations (Nerlove, 1958) however are afflicted with non-vanishing forecasting errors in the presence of persistent periodic fluctuations. The costs associated with repeated and systematic mistakes in forecasting prices makes both models of expectation formation implausible as a model of rational behavior because the firm has an incentive to gather information on the market structure. In fact, a simple diagram depicting supply versus market-clearing price for past periods suffices to obtain the required information.

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The rational expectations paradigm, Muth (1961), rules out any cyclical fluctuations in the Cobweb model on the basis of these considerations: Under rational expectations perceived and actual distribution of prices have to coincide, which is only possible in a steady state of the Cobweb model. One has to point out that strong assumptions underlie this reasoning. For instance, what convinces an agent to believe that the distribution of past prices have been used to determine the supply in every past period? The decision-maker in the firm has to have access to a complete record of historically perceived distribution of prices in order to decide that matter. Observed data however can be interpreted in many different ways and thus may support or "rationalize" many different supply decisions.

In the theory of rational beliefs, Kurz (1994,1997), rationality of agents is judged by the compatibility of their forecast with past data. In short, a rational belief is a subjective probability distribution that is compatible with the data in the sense that the empirical measures coincide. In the Cobweb model this means the probability of events under the agent's perceived (possibly non-stationary) and the actual distribution of the price—both averaged over time—have to be identical. We show that permanent (and in particular cyclic) price fluctuations can be reconciled with rational decisions in the Cobweb model even if they are not compatible with a rational expectations equilibrium.

2. Model. Assume there is one firm with cost function $C(q)$ that has to decide on total supply before knowing the future price of the good. If the firm acts as price-taker[1], the profit maximization problem is given by

$$\max_q p^e\, q - C(q) \tag{1}$$

where p^e is the expected price. If the forecasted price is a random variable then p^e is its expected value. Assuming a increasing, strictly concave and continuously differentiable cost function, there is an optimal output $q^\star = (C')^{-1}(p^e)$. The actual market-clearing price is then determined by

$$p^\star = D^{-1}(q^\star) = D^{-1}\left[(C')^{-1}(p^e)\right] =: F(p^e) \tag{2}$$

where demand $D(p)$ is assumed to be strictly decreasing in the price.

Under naive expectations the last observed price is taken as the forecast, i.e. $p^e = p(t-1)$. The dynamics of the price is then determined by the law of motion F:

$$p(t) = F(p(t-1)) \tag{3}$$

[1]An assumption made for simplicity of presentation only; one can also analyze a monopolistic firm.

Depending on the functional form of the law of motion F, the price dynamics can be governed by stable or unstable fixed points, cycles of any period, or chaotic dynamics, see e.g. Devaney (1989). Attention is restricted here to the case in which F permits a periodic solution of period $n > 1$ and a steady state.

3. Results. Suppose the law of motion F has a cycle of period $n \geq 2$ with values $p_1, ..., p_n$ (i.e. $F(p_i) = p_{i+1}$ for $i = 1, ..., n-1$, and $F(p_n) = p_1$), as well as a fixed point p_0 (i.e. $F(p_0) = p_0$). A firm forecasting at the first time of production the price $p(0) = p_1$ makes the output decision $q_1 = (C')^{-1}(p_1)$. This implies a market-clearing price $p(1) = D^{-1}(q_1) = F(p_1) = p_2$. In the next period, $q_2 = (C')^{-1}(p_2)$ implies the market-clearing price $p(2) = D^{-1}(q_2) = F(p_2) = p_3$. After n periods the cycle repeats. Analogously for any other initial price forecast $p(0) = p_i$. Thus naive expectations lead to the sequence of market-clearing prices $p(t) = p_{(i+t-1 \bmod n)+1}$. The forecast fails to correctly predict the price and thus we do not observe a rational expectations equilibrium. Since the law of motion F also possesses a fixed point, another sequence of prices can occur under naive expectations. If the very first forecast of the firm is based on $p(0) = p_0$, the output decision $q_0 = (C')^{-1}(p_0)$ implies the market-clearing price $p(1) = D^{-1}(q_0) = F(p_0) = p_0$. The firm's forecast is fulfilled. $p^e = p(t)$ and $p(0) = p_0$ constitutes a rational expectations equilibrium. Suppose the firm has all information needed to compare forecast with observed price. Then cyclic dynamics lead to an obvious pattern in forecasting errors while at the steady state forecast and actual price are identical. However in both cases the empirical distributions of forecast and actual price are identical. This observation is central to the theory of rational beliefs. Let us elaborate this point in more detail.

Suppose there is an infinite history of observations and that past prices are equal to p_i, $i = 1, .., n$, or p_0. Let the price p_0 occur with relative frequency α, and assume all prices p_i, $i = 1, .., n$, are observed with the same frequency $\beta := (1 - \alpha)/n$. The empirical measure of the past prices is given by $\mu = \alpha\, \delta_{p_0} + \beta \sum_{i=1}^{n} \delta_{p_i}$. If a firm constructs the empirical measure μ and believes a forecast of a particular price makes sense, picking either p_0, or any p_i, $i = 1, .., n$, is consistent with the history of the economy as long as the forecast resembles on average the empirical measure μ. Under such behavior the empirical measure will never change.[2] In this economy we observe a rational belief equilibrium: (i) the forecast is on average consistent with the

[2]In fact, assuming an infinite history makes the empirical measure insensitive to any change in finitely many periods in time. With finitely many data, the empirical measure may be more or less sensitive to changes depending on the number of observations.

empirical measure, and (ii) the production decision based on this forecast implies the empirical measure on actual market-clearing prices.

Can one defend this concept of rationality against the criticism of neglect of "obvious" forecasting errors? Would not any firm invest in market analysis to prevent the forecasting mistake that occurs with frequency $1-\alpha$? Consider a firm with a lifespan shorter than n periods (or which keeps records of forecasts for a time horizon not exceeding n). The firm will never observe the cycle because it does not have sufficient data to obtain the systematic mistake by comparing its forecast with the subsequently observed price over its lifetime. More generally one can state that avoiding forecast errors requires the firm to have sufficient knowledge on the structure of the economy. While this is possible in the simple model discussed here—provided the history of forecasts is known—, more elaborate models with several interdependent markets and exogenous shocks may render this task impossible even under complete information about own past forecasts.

Short lifetime of agents relative to the history of an economy is common to many rational belief models (Kurz, 1997), typically formulated as overlapping generations models. In the Cobweb model as well in this class of models the technical tool to implement a forecasting mechanism that generates a prescribed empirical measure is assessment variables. Let y_t be a stochastic process with values in $\{p_0, p_1, ..., p_n\}$ such that (almost surely) $\lim_{T\to\infty} 1/T \sum_{t=0}^{T} \delta_{y_t} = \mu$, μ the empirical measure of the Cobweb model introduced above. y_t is the assessment variable of the firm when making a forecast. Under rational beliefs only the empirical measure of y_t is prescribed while all other statistical properties are not restricted. Depending on the sample paths of the process y_t the price may exhibit, for instance, cyclic behavior, occasional switches between regimes of fixed or varying prices as well as non-stationary fluctuations. Each price dynamics that can be derived from an assessment variable y_t with empirical measure μ constitutes a rational belief equilibrium.

4. Conclusion. This note illustrated that permanent price fluctuations of the "cobweb type" can be reconciled with rational behavior when interpreting the model in the theory of rational beliefs, Kurz (1994). Starting from the empirically observed phenomenon that agents' forecasts exhibit a high degree of diversity even under identical information, the notion of rationality in this theory requires only compatibility of the forecast with historical data (in the sense that the empirical measures coincide). The Cobweb cycle discussed here provides the most simple example of a rational belief equilibrium that is not a rational expectations equilibrium.

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